# LIBERTY PAPER SET

**STD. 10 : Mathematics (Standard) [N-012(E)]**

**Full Solution**

**Time : 3 Hours** **ASSIGNMENT PAPER 7**

## Section-A

**1.** (D) Right angle **2.** (B) $\frac{4}{3}\pi^{4-}$ **3.** (D) 90° **4.** (C) $2\pi r\ (h + r)$ **5.** (D) 40 **6.** (A) $\frac{1}{81}$ **7.** 2 **8.** Similar **9.** $2x + 3y - 42 = 0$ **10.** $a + (n - 1)d$ **11.** 0 **12.** 2 **13.** False **14.** True **15.** True **16.** True **17.** Yes **18.** 10 **19.** Point of contact **20.** 6 **21.** (c) – 3 **22.** (a) 3 **23.** (b) $\frac{1}{\cot\theta}$ **24.** (a) $\frac{\cos\theta}{\sin\theta}$

## Section-B

**25.** $65 = 5 \times 13$

$169 = 13 \times 13$

$\therefore$ HCF (65, 169) = 13

**26.** $\frac{3x}{2} - \frac{5y}{3} = -2$

$\therefore 9x - 10y = -12$ ...(1)

$\therefore y = \frac{9x + 12}{10}$ ...(2)

$\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$

$\therefore 2x + 3y = 13$ ...(3)

Put value of equation (2) in equation (3)

$2x + 3y = 13$

$\therefore 2x + 3\left(\frac{9x + 12}{10}\right) = 13$

$\therefore 2x + \frac{27x + 36}{10} = 13$

$\therefore 20x + 27x + 36 = 130$

$\therefore 20x + 27x = 130 - 36$

$\therefore 47x = 94$

$\therefore x = 2$

Put $x = 2$ in equation (2),

$y = \frac{9x + 12}{10}$

$\therefore y = \frac{9(2) + 12}{10} = \frac{18 + 12}{10} = \frac{30}{10} = 3$

$\therefore y = 3$

Therefore, the solution is : $x = 2$, $y = 3$

**27.** Suppose, the first positive integer = $x$,

The second consecutive positive integer = $x + 1$

According to condition,

$(x)^2 + (x + 1)^2 = 365$

$\therefore\ x^2 + x^2 + 2x + 1 = 365$

$\therefore\ 2x^2 + 2x + 1 - 365 = 0$

$\therefore\ 2x^2 + 2x - 364 = 0$

$\therefore\ x^2 + x - 182 = 0$

$\therefore\ x^2 + 14x - 13x - 182 = 0$

$\therefore\ x(x + 14) - 13(x + 14) = 0$

$\therefore\ (x - 13)(x + 14) = 0$

$\therefore\ x - 13 = 0$ OR $x + 14 = 0$

$\therefore\ x = 13$ OR $x = -14$

But $x = -14$ is not positive integer, therefore, required two consecutive positive integers will be 13 and 14.

**28.** $\mathbf{x^2 - 3x - 10 = 0}$

$\therefore\ x^2 - 5x + 2x - 10 = 0$

$\therefore\ x(x - 5) + 2(x - 5) = 0$

$\therefore\ (x - 5)(x + 2) = 0$

$\therefore\ x - 5 = 0$ OR $x + 2 = 0$

$\therefore\ x = 5$ OR $x = -2$

$\therefore$ The roots of this equation : 5, – 2

**29.** Here $a = 2$, $d = 7 - 2 = 5$, $n = 15$

we have, $a_n = a + (n - 1)d$

$\therefore\ a_{15} = 2 + (15 - 1)(5)$

$\therefore\ a_{15} = 2 + (14)(5)$

$\therefore\ a_{15} = 2 + 70$

$\therefore\ a_{15} = 72$

So, the 15th term of an AP is 72.

**30.** $2\,cot^2 45° + sin^2 30° - cos^2 60°$

$$= 2(1)^2 + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2$$

$$= 2 + \frac{1}{4} - \frac{1}{4}$$

$$= 2$$

**31.** LHS $= (cosec\ \theta - cot\ \theta)^2$

$$= \left(\frac{1}{sin\ \theta} - \frac{cos\ \theta}{sin\ \theta}\right)^2$$

$$= \left(\frac{1 - cos\ \theta}{sin\ \theta}\right)^2$$

$$= \frac{(1 - cos\ \theta)^2}{sin^2\theta}$$

$$= \frac{(1 - cos\ \theta)^2}{1 - cos^2\theta}$$

$$= \frac{(1 - cos\ \theta)(1 - cos\ \theta)}{(1 - cos\ \theta)(1 + cos\ \theta)}$$

$$= \frac{1 - cos\ \theta}{1 + cos\ \theta} = \text{RHS}$$

**32.** We have, $\angle$ AOB + $\angle$ APB = 180°

$\therefore$ 135° + $\angle$ APB = 180°

$\therefore$ $\angle$ APB = 180° – 135°

$\angle$ APB = 45°

$\therefore$ $\angle$ OPB $= \frac{1}{2}$ $\angle$ APB

$= \frac{1}{2}$ 45°

$\angle$ OPB = 22.5°

**33.** Volume of cube = $x^3$

$\therefore$ 1000 = $x^3$

$\therefore$ $x^3 = 10^3$

$\therefore$ $x$ = 10 cm

$l = 2x = 2(10) = 20$ cm

$b = x = 10$ cm

$h = x = 10$ cm

CSA of cuboid

= 2 ($lb + bh + hl$)

= 2 [(20 × 10) + (10 × 10) + (10 × 20)]

= 2 [200 + 100 + 200]

= 2 [500]

= 1000 cm²

**34.** Here highest frequency is 16.

$\therefore$ Class = 3 – 5, $h$ = 2, $l$ = 3

$\therefore$ $f_1 = 16$

$f_0 = 14$

$f_2 = 4$

We have, Z $= l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

$= 3 + \left[\frac{16-14}{2 \times 16 - 14 - 4}\right] \times 2$

$= 3 + \frac{2 \times 2}{32 - 18}$

$= 3 + \frac{4}{14}$

$= 3 + \frac{2}{7}$

$= 3 + 0.29$

Z = 3.29

**35.** M + $\overline{x}$ = 44

M – $\overline{x}$ = 2

2M = 46

$\therefore$ M $= \frac{46}{2}$

$\therefore$ M = 23

M + $\overline{x}$ = 44

$\therefore$ 23 + $\overline{x}$ = 44

$\therefore$ $\overline{x}$ = 44 – 23

$\therefore$ $\overline{x}$ = 21

Z = 3M – 2$\overline{x}$

$\therefore$ Z = 3(23) – 2(21)

$\therefore$ Z = 69 – 42

$\therefore$ Z = 27

**36.** Total number of marbles = 5 + 8 + 4 = 17

Total number of marbles = 17

(i) Suppose event A is getting a red marble.

$\therefore$ P(A) = $\frac{\text{Number of red marbles}}{\text{Total number of marbles}}$

$\therefore$ P(A) = $\frac{5}{17}$

(ii) Suppose event B is not getting a green marble,

i.e. getting red or white marbles

$\therefore$ P(B) = $\frac{\text{Number of red and white marbles}}{\text{Total number of marbles}}$

$\therefore$ P(B) = $\frac{5+8}{17}$

$\therefore$ P(B) = $\frac{13}{17}$

**37.** A coin is tossed three times, then the result are HHH, HNT, HTH, THH, HTT, THT, TTH and TTT.

$\therefore$ Total numbers of Result = 8

(i) Suppose a Coin is tossed three times A be the event "at most one head is found"

There are 4 results HTT, THT, TTH and TTT.

$\therefore$ The numbers of outcomes favourable to A = 4

$\therefore$ P (A) = $\frac{4}{8}=\frac{1}{2}$

(ii) Suppose a Coin is tossed three times B be the event "the number of head is greater than the number of tails"

There are 4 results HHH, HHT, HTH and THH.

$\therefore$ The numbers of outcomes favourable to B = 4

$\therefore$ P (B) = $\frac{4}{8}=\frac{1}{2}$

## Section-C

**38.** Let, $x^2 - 7 = 0$

$\therefore$ $x^2 = 7$

$\therefore$ $x = \pm\sqrt{7}$

$\therefore$ $x = \sqrt{7}$ OR $x = -\sqrt{7}$

$\therefore$ $\alpha = \sqrt{7}$ OR $\beta = -\sqrt{7}$

$a = 1, b = 0, c = -7$

Sum of zeroes = $\alpha + \beta = \sqrt{7} - \sqrt{7} = 0 = \frac{-0}{1} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$

Product of zeroes = $\alpha \cdot \beta = \frac{c}{a}$ $(\sqrt{7})(-\sqrt{7}) = -7 = \frac{-7}{1} - \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x2}$

**39.** We have,

$\alpha + \beta = -\frac{b}{a}$ $\qquad$ $\alpha \cdot \beta = \frac{c}{a}$

$\therefore$ $9 = -\frac{b}{a}$ $\qquad$ $14 = \frac{c}{a}$

$\therefore$ $\frac{9}{1} = -\frac{b}{a}$ $\qquad$ $\frac{14}{1} = \frac{c}{a}$

$\therefore$ $a = 1, b = -9, c = 14$

So, one quardratic polynomial which fits the given conditions is $x^2 - 9x + 14$. you can check that any other quadratic polynomial which fits these conditions will be of the form $k(x^2 - 9x + 14)$, where $k$ is real.

**40.** Here, $a = 16$, $d = 6 - 16 = -10$, $n = 30$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{30} = \frac{30}{2}[2(16) + (30-1)(-10)]$

$\therefore \quad S_{30} = 15\,[32 + 29\,(-10)]$

$\therefore \quad S_{30} = 15\,(32 - 290)$

$\therefore \quad S_{30} = 15\,(-258)$

$\therefore \quad S_{30} = -3870$

So, the sum of first 30 terms will be – 3870.

**41.** Here, $S_7 = 49$, $S_{17} = 289$, $S_n =$ ______

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore\ S_7 = \frac{7}{2}[2a + (7-1)d]$

$\therefore\ 49 = \frac{7}{2}(2a + 6d)$

$\therefore\ 49 = 7\,(a + 3d)$

$\therefore\ a + 3d = 7$ ...(1)

Same as, $S_{17} = \frac{17}{2}[2a + (17-1)d]$

$\therefore\ 289 = \frac{17}{2}(2a + 16d)$

$\therefore\ 289 = 17\,(a + 8d)$

$\therefore\ a + 8d = 17$ ...(2)

Subtracting equation (2) from equation (1),

$(a + 3d) - (a + 8d) = 7 - 17$

$\therefore \quad a + 3d - a - 8d = -10$

$\therefore \quad -5d = -10$

$\therefore \quad d = 2$

Put $d = 2$ in equation (1),

$a + 3d = 7$

$\therefore\ a + 3(2) = 7$

$\therefore\ a + 6 = 7$

$\therefore\ a = 1$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2(1) + (n-1)2]$

$= n\,(1 + n - 1)$

$= n\,(n)$

$\therefore\ S_n = n^2$

**42.** Suppose, A (4, –1) and B (–2, –3) connecting the line segment AB are the trisection points P and Q.

$\therefore$ AP = PQ = QB

Here, point P divides AB internally in ratio 1 : 2.

$\therefore$ The co-ordinate of point

$$P = \left(\frac{1(-2)+2(4)}{1+2}, \frac{1(-3)+2(-1)}{1+2}\right)$$

$$= \left(2, -\frac{5}{3}\right)$$

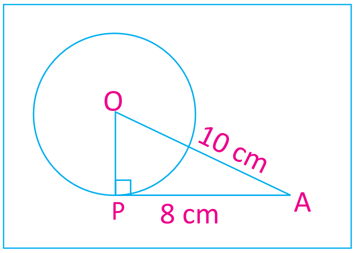
Same as, point Q divides AB in ratio 2 : 1.

$\therefore$ The co-ordinate of point

$$Q = \left(\frac{2(-2)+1(4)}{2+1}, \frac{2(-3)+1(-1)}{2+1}\right)$$

$$= \left(0, -\frac{7}{3}\right)$$

**43.**




Here OA = 10 cm  PA = 8 cm

$\overline{OP} \perp \overline{PA}$  $\therefore \angle OPA = 90°$

So from $\Delta$OPA,

$OP^2 + PA^2 = OA^2$

$R^2 + 8^2 = 10^2$

$R^2 + 64 = 100$

$R^2 = 100 - 64$

$R^2 = 36$

R = 6 cm

$\therefore$ Diameter = 2R

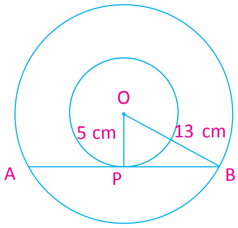
= 2(6)

$\therefore$ D = 12 cm

**44.** We have, $r_1$ = 13 cm

$r_2$ = 5 cm

$\overline{OP} \perp \overline{PB}$




$\therefore$ In $\Delta$ OPB,

$OP^2 + PB^2 = OB^2$

$5^2 + PB^2 = 13^2$

$25^2 + PB^2 = 169$

$PB^2 = 169 - 25$

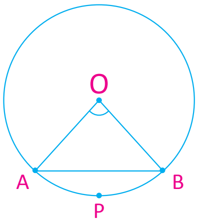
$PB = \sqrt{144}$

PB = 12 cm

Length of chord = 2PB = 2(12) = 24 cm

**45.** Here we have, $d = 42$ cm

$\therefore \quad r = \frac{d}{2} = \frac{42}{2} = 21$ cm $\quad \theta = 60°$




(a) Area of minor sector OAPB

$= \frac{\pi r^2 \theta}{360}$

$= \frac{22 \times 21 \times 21 \times 60}{7 \times 360}$

$= \frac{11 \times 2 \times 21 \times 7 \times 3 \times 60}{7 \times 60 \times 3 \times 2}$

$= 11 \times 21$

$= 231 \text{ cm}^2$

(b) Here $\theta = 60°$ so $\Delta$ OAB must be an equilateral triangle.

So if OA = OB = 21 cm

$\therefore$ AB = 21 cm

Area of minor segment APB

= Area of mnior OAPB – Area of $\Delta$ AOB

$= 231 - \frac{\sqrt{3}}{4} \text{ side}^2$

$= 231 - \left(\frac{\sqrt{3}}{4} \times 21^2\right)$

$= 231 - \frac{441 \times \sqrt{3}}{4}$

$= \left(231 - \frac{441\sqrt{3}}{4}\right) \text{ cm}^2$

**46.** A box contains 90 discs which are numberd from 1 to 90.

Total number of discs = 90

(i) Suppose event A of drawing a two digit number of discs (10 to 90 = 81).

$\therefore \text{ P(A)} = \frac{\text{Total number of two digit number}}{\text{Total number of outcomes}}$

$= \frac{81}{90}$

$\therefore \text{ P(A)} = \frac{9}{10}$

(ii) Suppose event B of drawing a perfect square (1, 4, 9, 16, 25, 36, 49, 64, 81 = 9).

$\therefore \text{ P(B)} = \frac{\text{Total number of perfect square}}{\text{Total number of outcomes}}$

$= \frac{9}{90}$

$\therefore \text{ P(B)} = \frac{1}{10}$

(iii) Suppose event C of drawing a number divisible by 5 (5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90 = 18).

$\therefore \text{ P(C)} = \frac{\text{Total number of divisible by 5}}{\text{Total number of outcomes}}$

$\therefore \text{ P(C)} = \frac{18}{90}$

$\therefore \text{ P(C)} = \frac{1}{5}$

## Section-D

**47.** Suppose, the numerator is $x$ and the denominator is $y$.

the fraction = $\frac{x}{y}$

According to the first condition;

$$\frac{x+1}{y-1} = 1$$

$\therefore\ x + 1 = y - 1$

$\therefore\ x - y = -2$ ...(1)

According to the second condition;

$$\frac{x}{y+1} = \frac{1}{2}$$

$\therefore\ 2x = y + 1$

$\therefore\ 2x - y = 1$ ...(2)

Subtracting equation (1) and (2),

$$\begin{array}{r} x - y = -2 \\ 2x - y = 1 \\ - \quad + \qquad - \\ \hline \end{array}$$

$\therefore\ -x = -3$

$\therefore\ x = 3$

Put $x = 3$ in equation (1)

$x - y = -2$

$\therefore\ 3 - y = -2$

$\therefore\ y = 5$

Hence, the fraction = $\frac{x}{y} = \frac{3}{5}$

**48.** Given that,

$2x + 3y = 11$ ...(1)

$x - 2y = -12$ ...(2)

Lets multiply equation (2) by 2 and subtract from equation (1),

$$\begin{array}{r} 2x + 3y = 11 \\ 2x - 4y = -24 \\ - \quad + \qquad + \\ \hline \end{array}$$

$\therefore\ 7y = 35$

$\therefore\ y = \frac{35}{7}$

$\therefore\ y = 5$

Put $y = 5$ in equation (1),

$2x + 3(5) = 11$

$2x + 15 = 11$

$2x = 11 - 15$

$2x = -4$

$x = \frac{4}{2}$

$x = -2$

$\therefore$ Put $x = -2$ and $y = 5$

in, $y = kx + 9$

$\therefore\ 5 = k(-2) + 9$

$\therefore\ 5 - 9 = -2k$

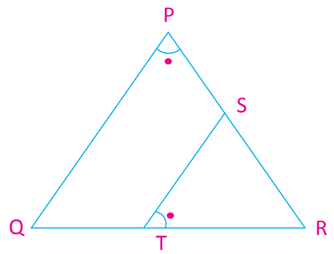
$\therefore\ -4 = -2k$

$\therefore\ k = \frac{-4}{2}$

$\therefore\ k = 2$

**49.** **Given :** S and T are points on sides PR and QR of $\Delta$ PQR such that $\angle P = \angle RTS$.

**To Prove :** $\Delta$ RPQ ~ $\Delta$ RTS




**Proof :** $\angle P = \angle RTS$ (Given)

$\therefore$ $\angle RPQ = \angle RTS$

In $\Delta$ RPQ and $\Delta$ RTS,

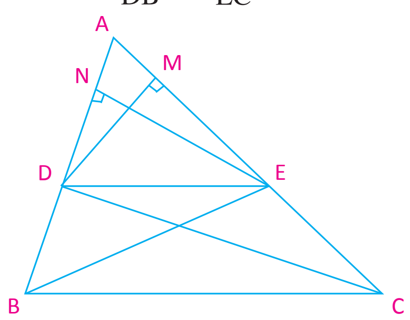
$\angle RPQ = \angle RTS$

$\angle PRQ = \angle SRT$ (Common angles)

$\therefore$ $\Delta$ RPQ ~ $\Delta$ RTS (AA similarity criterion)

**50.** Given: In ABC, a line parallel to side BC intersects AB and AC at D and E respectively.

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$




Proof: Join BE and CD and also draw DM $\perp$ AC and EN $\perp$ AB.

Then, ADE $= \frac{1}{2} \times AD \times EN$,

BDE $= \frac{1}{2} \times DB \times EN$,

ADE $= \frac{1}{2} \times AE \times DM$ and

DEC $= \frac{1}{2} \times EC \times DM$.

$$\therefore \frac{ADE}{BDE} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB} \quad ...(1)$$

and $$\frac{ADE}{DEC} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \quad ...(2)$$

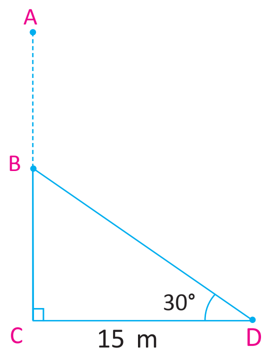
Now, $\Delta$ BDE and $\Delta$ DEC are triangles on the same base DE and between the parallel BC and DE.

then, BDE = DEC ...(3)

Hence from eq$^n$. (1), (2) and (3),

$$\frac{AD}{DB} = \frac{AE}{EC}$$

**51.**




Here, actual height of tree is AC - BD, bend part is BD.

In $\Delta ABC$, $\angle C = 90°$

$$\tan D = \frac{BC}{CD}$$

$$\therefore \quad \tan 30° = \frac{BC}{15}$$

$$\therefore \quad \tan 30° \times 15 = BC$$

$$\therefore \quad BC = \frac{1}{\sqrt{3}} \times 15$$

$$\therefore \quad BC = \frac{5 \times 3}{\sqrt{3}}$$

$$\therefore \quad BC = \frac{5 \times \sqrt{3} \times \sqrt{3}}{\sqrt{3}}$$

$$\therefore \quad BC = 5\sqrt{3} \text{ m}$$

Now, AB = BD

$$\text{let, } \sec 30° = \frac{BD}{CD}$$

$$\therefore \quad \frac{2}{\sqrt{3}} = \frac{BD}{15}$$

$$\therefore \quad DB = \frac{15 \times 2}{\sqrt{3}}$$

$$\therefore \quad DB = \frac{5 \times \sqrt{3} \times \sqrt{3} \times 2}{\sqrt{3}}$$

$$\therefore \quad DB = 10\sqrt{3} \text{ m}$$

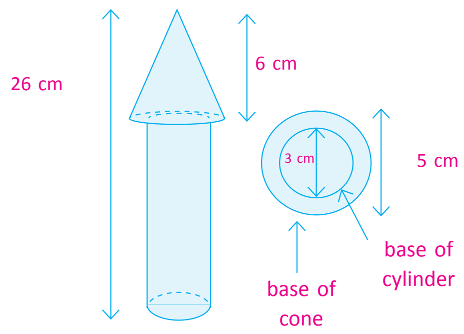
$$\therefore \quad AB = 10\sqrt{3} \text{ m}$$

Total height AC = AB + BC

$$= 10\sqrt{3} + 5\sqrt{3}$$

$$= 15\sqrt{3} \text{ m}$$

**52.**



Total height of the rocket = 26 cm

∴ Height of the cone + height of the cylinder = 26 cm

∴ 6 cm + Height of the cylinder = 26 cm

∴ Height of the cylinder = 20 cm

| Cone | Cylinder |
|---|---|
| $d = 5$ cm | $d = 3$ cm |
| ∴ $r = \frac{5}{2} = 2.5$ cm | ∴ $R = \frac{3}{2} = 1.5$ cm |
| ∴ $h = 6$ cm | $H = 20$ cm |

$l = \sqrt{r^2 + h^2}$

∴ $l = \sqrt{(2.5)^2 + (6)^2}$

∴ $l = \sqrt{6.25 + 36}$

∴ $l = \sqrt{42.25}$

∴ $l = 6.5$ cm

So, the area to be painted orange

= TSA of the cone – Base area of the cylinder

$= \pi r(l + r) - \pi R^2$

$= [3.14 \times 2.5 \times (6.5 + 2.5)] - (3.14 \times 1.5 \times 1.5)$

$= (3.14 \times 2.5 \times 9) - (3.14 \times 1.5 \times 1.5)$

$= 70.65 - 7.065$

$= 63.585$ cm$^2$

Now, the area to be painted yellow

= CSA of the cylinder + Area of one base of the cylinder

$= 2\pi RH + \pi R^2$

$= \pi R(2H + R)$

$= 3.14 \times 1.5 \times [2(20) + 1.5]$

$= 3.14 \times 1.5 \times 41.5 = 195.465$ cm$^2$

**53.**

| Cylindrical glass | Hemisphere |
|---|---|
| $d = 5$ cm. | $d = 5$ cm |
| ∴ $r = \frac{5}{2}$ cm | $r = \frac{5}{2}$ cm |
| ∴ $h = 10$ cm | |

The Apparent capacity of the glass $= \pi r^2 h$

$= 3.14 \times \frac{5}{2} \times \frac{5}{2} \times 10$

$= 1.57 \times 5 \times 5 \times 5$

$= 196.25$ cm$^3$

Volume of hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times 3.14 \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2}$

$= 32.71$ cm$^3$

So, the actual capacity of the glass

= Apparent capacity glass – Volume of the hemisphere

$= 196.25 - 32.71$

$= 163.54$ cm$^3$

**54.**

| Weight (in kg) | Number of students ($f_i$) | $cf$ |
|---|---|---|
| 40 – 45 | 2 | 2 |
| 45 – 50 | 3 | 5 |
| 50 – 55 | 8 | 13 |
| 55 – 60 | 6 | 19 |
| 60 – 65 | 6 | 25 |
| 65 – 70 | 3 | 28 |
| 70 – 75 | 2 | 30 |
| | $n = 30$ | |

Here, $n = 30$

$\therefore$ Since the 15$^{th}$ observation is contained in class 55-60, the median class is 55-60.

Then, $l$ = lower limit of median class = 55

$cf$ = Cumulative frequency of class preceding the median class = 13

$f$ = frequency of median class = 6

$h$ = class size = 5

$$\text{Median M} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times \text{h}$$

$$\therefore \text{M} = 55 + \left(\frac{15 - 13}{6}\right) \times 5$$

$$\therefore \text{M} = 55 + \frac{2 \times 5}{6}$$

$$\therefore \text{M} = 55 + 1.67$$

$$\therefore \text{M} = 56.67$$

So, median weight is 56.67 kg.